

# Cambridge O Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 2 6 6 4 1 2 0 9 2 1

#### **ADDITIONAL MATHEMATICS**

4037/11

Paper 1 May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

#### **INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Any blank pages are indicated.

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#### Mathematical Formulae

#### 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n}$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ 

Arithmetic series  $u_n = a + (n-1)d$ 

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series 1

$$u_n = ar^{n-1}$$
 $S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1)$ 
 $S_{\infty} = \frac{a}{1-r} \ (|r| < 1)$ 

#### 2. TRIGONOMETRY

*Identities* 

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 Find constants a, b and c such that  $\frac{\sqrt{p}q^{\frac{2}{3}}r^{-3}}{\left(pq^{-1}\right)^{2}r^{-1}} = p^{a}q^{b}r^{c}.$  [3]

2	A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time
	t seconds, $t \ge 0$ , is given by $s = (1+3t)^{-\frac{1}{2}}$ .

(a) Find the exact speed of the particle when t = 1. [3]

(b) Show that the acceleration of the particle will never be zero. [2]

- 3 A function f is such that  $f(x) = \ln(2x+1)$ , for  $x > -\frac{1}{2}$ .
  - (a) Write down the range of f.

[1]

A function g is such that g(x) = 5x - 7, for  $x \in \mathbb{R}$ .

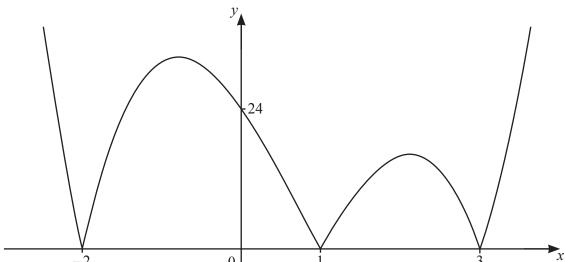
**(b)** Find the exact solution of the equation gf(x) = 13.

[3]

(c) Find the solution of the equation  $f'(x) = g^{-1}(x)$ .

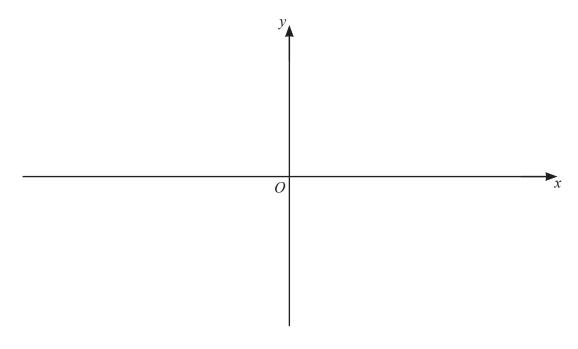
[6]

4 (a)



The diagram shows the graph of y = |f(x)|, where f(x) is a cubic. Find the possible expressions for f(x).

(b) (i) On the axes below, sketch the graph of y = |2x+1| and the graph of y = |4(x-1)|, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



(ii) Find the exact solutions of the equation |2x+1| = |4(x-1)|.

[4]

5 (a) Find the vector which is in the opposite direction to  $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$  and has a magnitude of 8.5. [2]

**(b)** Find the values of a and b such that 
$$5\binom{3a}{b} + \binom{2a+1}{2} = 6\binom{b+a}{2}$$
. [3]

6 (a) Write down the values of k for which the line y = k is a tangent to the curve  $y = 4\sin\left(x + \frac{\pi}{4}\right) + 10$ .

**(b)** (i) Show that 
$$\frac{1+\tan\theta}{1-\cos\theta} + \frac{1-\tan\theta}{1+\cos\theta} = \frac{2(1+\sin\theta)}{\sin^2\theta}.$$
 [4]

(ii) Hence solve the equation 
$$\frac{1 + \tan \theta}{1 - \cos \theta} + \frac{1 - \tan \theta}{1 + \cos \theta} = 3$$
, for  $0^{\circ} \le \theta \le 360^{\circ}$ . [4]

7 **(a)** The first three terms of an arithmetic progression are 1g3, 31g3, 51g3. Given that the sum to *n* terms of this progression can be written as 256 lg 81, find the value of *n*. [5]

## (b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are  $\ln 256$ ,  $\ln 16$ ,  $\ln 4$ . Find the sum to infinity of this progression, giving your answer in the form  $p \ln 2$ . [4]

## 8 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the exact coordinates of the points of intersection of the curve  $y = x^2 + 2\sqrt{5}x - 20$  and the line  $y = 3\sqrt{5}x + 10$ .

**(b)** It is given that  $\tan \theta = \frac{\sqrt{3} - 1}{2 + \sqrt{3}}$ , for  $0 < \theta < \frac{\pi}{2}$ . Find  $\csc^2 \theta$  in the form  $a + b\sqrt{3}$ , where a and b are constants.

9	A circle, centre $O$ and radius $r$ cm, has a sector $OAB$ of fixed area $10 \text{ cm}^2$ . Angle $AOB$ is $\theta$ radians and the perimeter of the sector is $P$ cm.							
	(a)	Find an expression for $P$ in terms of $r$ .	[3]					
	(b)	Find the value of $r$ for which $P$ has a stationary value.	[3]					
	(c)	Determine the nature of this stationary value.	[2]					
	(d)	Find the value of $\theta$ at this stationary value.	[1]					

10 The normal to the curve  $y = \tan\left(3x + \frac{\pi}{2}\right)$  at the point *P* with coordinates (p, -1), where 0 , meets the*x*-axis at the point*A*and the*y*-axis at the point*B*. Find the exact coordinates of the mid-point of*AB*.

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